TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Description Logic

Summer Semester 2022

Exercise Sheet 4

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Prof. Dr.-Ing. Franz Baader, Dr. Oliver Fernández Gil

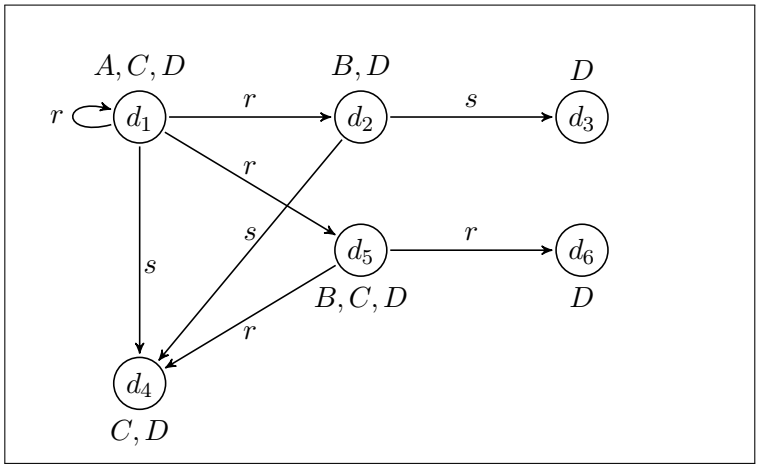
Exercise 4.1 Prove or refute the following claim: If an $\mathcal{ALC}$-concept description C is satisfiable w.r.t. an $\mathcal{ALC}$-TBox $\mathcal{T}$, then for all $n \geq 1$ there is a finite model $\mathcal{I}_n$ of $\mathcal{T}$ such that $|C^{\mathcal{I}_n}| \geq n$.

Does the claim hold true if the condition "$|C^{\mathcal{I}_n}| \geq n$" is replaced by "$|C^{\mathcal{I}_n}| = n$"?

Exercise 4.2 Consider the set

$$S := \{\exists s.C,\ \ \forall r.D,\ \ \exists s.D,\ \ C,\ \ D,\ \ A\},$$

and the following interpretation $\mathcal{I}$:

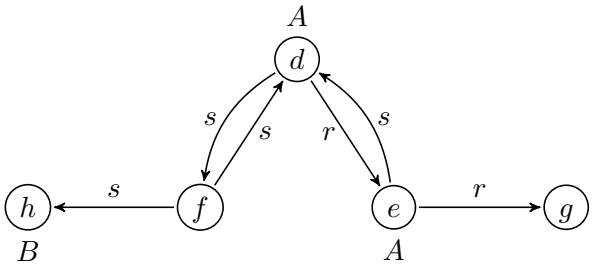


(a) Compute the S-types of each element in $\Delta^{\mathcal{I}}$ and the equivalence relation $\simeq_S$ on $\Delta^{\mathcal{I}}$.

(b) Compute the S-filtration of $\mathcal{I}$.

Exercise 4.3 Prove or refute the following claim: Given an $\mathcal{ALC}$-concept description C and an $\mathcal{ALC}$-TBox $\mathcal{T}$, if $\mathcal{I}$ is an interpretation and $\mathcal{J}$ its filtration w.r.t. $\mathsf{sub}(C) \cup \mathsf{sub}(\mathcal{T})$ (see Definition 3.14), then the relation $\rho := \{(d, [d]) \mid d \in \Delta^{\mathcal{I}}\}$ is a bisimulation between $\mathcal{I}$ and $\mathcal{J}$.

Exercise 4.4 For the following interpretation $\mathcal{I}$, draw the unraveling of $\mathcal{I}$ at d up to depth 5, i.e., restricted to d-paths of length at most 5 (see Definition 3.21):



Exercise 4.5 Prove or refute the following claim: If $\mathcal{K}$ is an $\mathcal{ALC}$-knowledge base and C is an $\mathcal{ALC}$-concept description such that C is satisfiable w.r.t. $\mathcal{K}$, then C has a tree model w.r.t. $\mathcal{K}$.